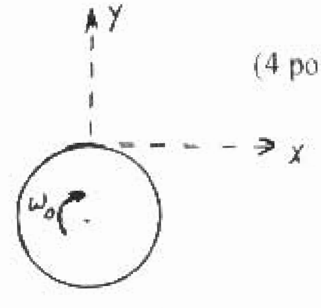
1998 US Physics Team, Exam 1
Solutions and Grading Key, Creative Response

Concerning double jeporady: Some problems require answers to be used in subsequent parts. Incorrect *answers* should be penalized only once. Correct *reasoning* should be acknowledged.

1. (25 points)

(a. 4) $\mathbf{v}_o = R\omega_o\,\mathbf{i}$ (in terms of the coordinate system shown) (4 points)



(b. 7) This is a projectile problem: with $t = 0$ at the top of the disc, we have

$x = v_o t = R\omega_o t$ (2)

and $y = -\frac{1}{2}gt^2$. (2)

At point B, $y = -R$, so that $t = \sqrt{(2R/g)}$ (2)

and therefore $x = R\omega_o\sqrt{(2R/g)}$ (1)

(c. 7) One may use either conservation of energy, or projectile kinematics.

Energy conservation approach:

$E_A = E_B$	(recognizing the principle)	(3)
$\frac{1}{2}m(R\omega_o)^2 + mgR = \frac{1}{2}mv_B^2 + 0$	(applying it)	(2)
$v_B = [(R\omega_o)^2 + 2gR]^{1/2}$	(solving)	(2)

Projectile kinematics approach:

$v_B^2 = v_x^2 + v_y^2$, (2)

where $v_x = R\omega_o$ (2)

and $v_y^2 = 2gR$ [from $v_y^2 = v_{oy}^2 + 2a_y(y - y_o)$] (2)

and thus, $v_B = [(R\omega_o)^2 + 2gR]^{1/2}$ (1)

(d. 7) By conservation of angular momentum,

$L = L'$	(recognizing the principle)	(2)
which becomes $I_o\omega_o = I'\omega'$	(applying it)	(1)

Let's calculate I': it has two contributions, one for the damaged disc missing the chip, and one for the chip: When the chip comes loose,

$I' = [\,I_o - mR^2\,]$(for the damaged disc) + $[\,mR^2\,]$(for the chip)

$= I_o$. (3)

Hence, $\omega_o = \omega'$. (1)

2. (25 points)

(a, 5) **F** = m**a** becomes $kx_o - mg = 0$ (3)

so that $k = mg / x_o = (1\text{ kg})(9.8\text{ N/kg}) / (0.2\text{ m}) = 49$ N/m. (2)

(b, 10) One may use the work-energy theorem,

$W_{nc} = \Delta E$ (3)

where W_{nc} is the work done by non-conservative forces and E is the total mechanical energy. With the zero of gravitational P.E. at the initial position of the hanging mass, $W_{nc} = \Delta E$ becomes

$-\mu mgx = \frac{1}{2}kx^2 - mgx$ (4)

The $x \neq 0$ solution gives $x = 2(mg/k)(1-\mu) = 0.3$ m. (3)

(c, 10) The system is equivalent to a particle of mass $2m$ being pulled in one direction by the force mg in the opposite direction by the force kx. Thus, Newton's second law gives

$mg - kx = 2ma$ (3)

which says $a + (k/2m)x = \frac{1}{2}g$, which is the equation of motion of a simple harmonic oscillator (in a uniform gravitational field), with angular frequency $\omega = \sqrt{(k/2m)}$. (4)

[Alternatively, if the student notes that the angular frequency squared equals k divided by the mass that is oscillating, and notes that $2m$ is the mass that oscillates, give full credit for part (c) up to here.]

Next, convert to period: $T = 2\pi/\omega$ gives $T = 2\pi\sqrt{(2m/k)} = 1.27$ s. (3)

3. (25 points)

(a, 9) One may apply conservation of energy, $E_i = E_f$, (2)

and obtain (choosing P.E. = 0 at $\theta = 0$):

$(2m)g(2L) = [\frac{1}{2}mv_1^2 + mg(2L - L\cos\theta)] + [\frac{1}{2}mv_2^2 + mg(2L)(1-\cos\theta)]$

which simplifies to

$0 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 - 3mgL\cos\theta$. (3)

We note that for all θ, $v_1 = L\omega$ and $v_2 = (2L)\omega = 2v_1$. (2)

Hence, $v_1 = [(6/5)gL\cos\theta]^{1/2}$ and $v_2 = 2[(6/5)gL\cos\theta]^{1/2}$ (1)

so that, at $\theta = 0$, $v_1 = [(6/5)gL]^{1/2}$ and $v_2 = 2[(6/5)gL]^{1/2}$ (1)

(b, 16)

Consider the bottom mass (mass #2) [6 points to be earned here]:

The radial component of **F** = m**a** becomes

$T_2 - mg\cos\theta = m\,v_2^2/2L$ (3)

Using the result $v_2^2 = (24/5)gL\cos\theta$ from part (a), one finds

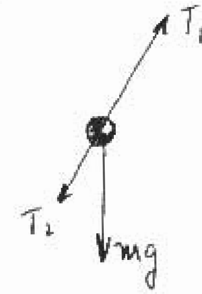
$T_2 = (17/5)mg\cos\theta$ (2)

which is $< 4.2mg$ for all θ, so the lower rod does not break. (1)

[Note: if the student says "we need consider only the top rod because if either rod breaks it must be the top one," award the (1) point for this

conclusion; however, the the student will still need to find
T_2 as a function of θ in order to determine the angle where the upper rod breaks.]

Consider next the upper mass (mass #1) [10 points to be earned here]:
The radial component of **F** = ma becomes
$T_1 - T_2 - mg\cos\theta = mv_1^2/L$ (5)
Using $v_1^2 = (6/5)\,gL\cos\theta$ and $T_2 = (17/5)\,mg\cos\theta$, one obtains
$T_1 = (28/5)\,mg\cos\theta$. (3)
Thus, T_1 will equal $4.2mg$ when $\theta = 41.4°$ (2)



4. (25 points)

(a, 7) **F** = m**a** applied to the satellite gives
$GMm/r^2 = mv^2/r$ (3)
where for the satellite, $v = 2\pi r/T$. (3)
Thus one obtains $T^2 = (4\pi^2/GM)\,r^3$,
so that $C = 4\pi^2/GM$. (1)

(b, 6) For geosynchronous orbit, $T = 24$ hrs. $= 8.64\times10^4$ s. (3)
Thus Kepler's third law and the Earth data give $r = 4.22\times10^7$ m. (3)

(c, 6) Let $T_o = 8.64\times10^4$ s, and let $r_o = 4.22\times10^7$ m, i.e, let the subscript denote the geosynchronous orbit data. With the change in orbit radius, we have
$r = r_o + \Delta r$ and $T = T_o + \Delta T$, so Kepler's third law becomes
$T_o^2(1 + \Delta T/T_o)^2 = C\,r_o^3\,(1 + \Delta r/r_o)^3$. (1)
Use the binomial expansion to obtain
$\Delta T = (3/2)\,(T_o/r_o)\,\Delta r$. (3)
With $\Delta r = -2$ km, it follows that $\Delta T = -6.1$ s (2)
[Of these last two points, one point is for the 6.1 s; one point is for either including the minus sign or stating that the satellite's period is *shorter* than before.]

(d, 6) Both the satellite and observer are revolving about the Earth's center of mass.
Let v_s = speed of the satellite relative to Earth's center of mass;
Let v_o = speed of the observer relative to Earth's center of mass.
The quantity we seek is $v_s - v_o$. Thus,
$v_s - v_o = R_e\,(\omega_s - \omega_o)$
$= 2\pi R_e\,(1/T_s - 1/T_o)$ (2)
Writing $T_s = T_o\,(1 + \Delta T/T_o)$ and using the binomial expansion, one finds
$v_s - v_o = -(2\pi R_e/T_o^2)\,\Delta T$. (2)
Numerically, $v_s - v_o = -2\pi(6.37\times10^6\text{ m})(8.64\times10^4\text{ s})^{-2}(-6.1\text{ s})$
$= 3.3$ cm/s. (1)
Because the Earth rotates from west to east, and because the satellite completes one "lap" about the Earth's CM in 6.1 s less than the observer, the spot moves *east* of the observer at the speed of 3.3 cm/s. (1)